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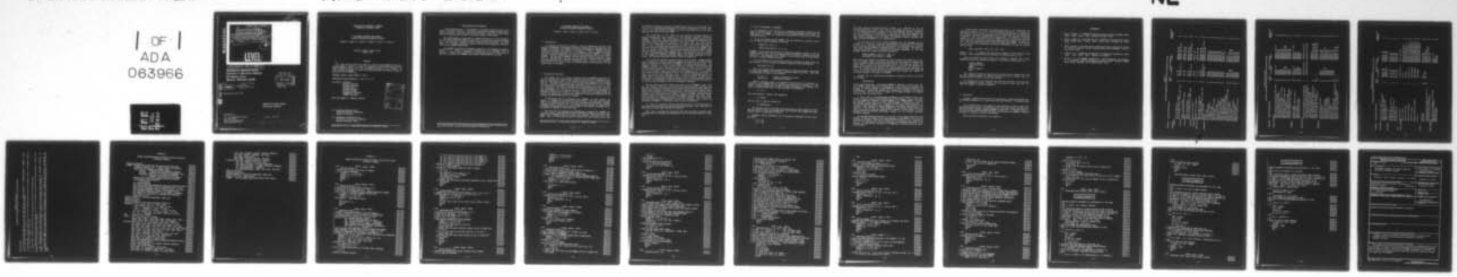

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# AN AUGMENT INTERFACE FOR BRENT'S MULTIPLE PRECISION ARITHMETIC PACKAGE

Richard P. Brent (1), Judith A. Hooper (2), and J. M. Yohe (2)

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## ABSTRACT

We describe the procedure required to interface the FORTRAN multiple precision package of Richard P. Brent (as described in ACM Transactions on Mathematical Software, March, 1978) with the AUGMENT precompiler for FORTRAN. We also indicate the method of using the multiple precision arithmetic package in conjunction with AUGMENT.

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Key words: Arithmetic
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## SIGNIFICANCE AND EXPLANATION

In some applications, it is necessary to use higher precision than is afforded by standard software. The multiple precision arithmetic package developed by Richard P. Brent and described in the March, 1978 issue of ACM Transactions on Mathematical Software is extremely useful in such cases.

The disadvantages of using Brent's package directly are (1) the difficulty of converting existing programs to make use of the multiple precision package, and (2) the fact that in order to write a program using the package, one must parse the arithmetic expressions oneself and write the program as a series of calls on the package subroutines.

The AUGMENT precompiler for FORTRAN, developed at the Mathematics Research Center by F. D. Crary, is designed to simplify the use of packages such as Brent's. In this report, we describe the necessary interface to enable one to use Brent's package with AUGMENT, and provide instructions for its use.

# AN AUGMENT INTERFACE FOR BRENT'S MULTIPLE PRECISION ARITHMETIC PACKAGE

Richard P. Brent, Judith A. Hooper, and J. M. Yohe

## 1. Introduction:

The purpose of this note is twofold: first, we demonstrate the ease with which a well-designed nonstandard arithmetic package may be interfaced with the AUGMENT precompiler for FORTRAN [4]; second, we provide an interface and user instructions to enable the reader to use Richard P. Brent's FORTRAN multiple precision arithmetic package [1], [2] in conjunction with AUGMENT. This makes the use of Brent's package far more natural and convenient than its use without AUGMENT. With the aid of AUGMENT, the user declares multiple precision variables as type MULTIPLE, and then, for the most part, simply writes the program as though MULTIPLE were a standard FORTRAN data type. In only a few instances must the user write explicit calls on package modules; these cases will be discussed later in the paper.

## 2. Writing the interface:

We assume that the reader is familiar with the AUGMENT precompiler, at least to the extent of knowing what is meant by such terms as "supporting package" and "description deck". This degree of familiarity may be gained by reading [4]. The supporting package to be interfaced with AUGMENT is the FORTRAN multiple precision arithmetic package described by Brent in [1] and [2]. This is a collection of portable subroutines which performs not only basic arithmetic operations, but also all of the ANSI standard mathematical functions and many nonstandard ones, in multiple precision. The precision of the package is governed entirely by the user at run time, and may even be changed during the course of a computation, provided the dimensions of the arrays reserved for the multiple precision numbers are not exceeded.

In interfacing this or any package with AUGMENT, we must specify the amount of storage to be allocated to each variable. This will place an upper limit on the operating precision of the multiple precision arithmetic package, although nothing prevents one from using a lower precision in computations. Increasing the precision beyond that provided in this standard interface is not difficult; we address this question later.

The first step in interfacing the package was to prepare the AUGMENT description decks. The multiple precision arithmetic package (although not designed with an AUGMENT interface in mind) was extremely compatible with AUGMENT: most of the multiple precision routines were cast as subroutines, with

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the numbers of arguments expected by AUGMENT (and in the order expected); nearly all of the manipulations required for a complete package were already provided (in the form assumed by AUGMENT); and all of the subroutines in the package bore the prefix MP in their names.

The preparation of the description deck therefore proceeded easily; we simply went down the list of routines in the multiple precision arithmetic package, associating them when possible with standard FORTRAN operations and functions. When such a natural association was not possible, we assigned function names (usually obtained by dropping the prefix 'MP' from the routine name). The description of each routine was coded as per the instructions in [5]. In only a few cases were we unable to do this: most of the input/output routines and error checking routines could not be interfaced with AUGMENT (they must be called explicitly), and the routines which provide constants needed special attention, as we shall discuss below. Routines which did not conform to the usual expectations of AUGMENT, such as the routine to add the quotient of two integers to a multiple precision number, were simply described as functions. The resulting description deck is shown in Appendix B.

The routines to generate constants posed a small problem: AUGMENT assumes that routines will have at least one argument in addition to the result (this might be regarded as a deficiency in AUGMENT), and these routines did not. We therefore decided to write a short routine to interface these routines with AUGMENT, casting it as a conversion routine which "converts" the Hollerith name of the desired constant to the value of the constant. This routine is called with the Hollerith name of the constant as an argument (e.g., 'PI'), unpacks this Hollerith string, determines which of the constant-generating routines to call, and returns the resulting value. Once this routine was written, it seemed logical to include the capability of run-time conversion of numeric constants, so we extended the routine by adding a call to another package routine to convert the (presumably numeric) Hollerith string to multiple precision if it did not match the name of any of the "standard" constants.

We also wrote six trivial logical functions to allow AUGMENT to deal with the six logical operators in the context of multiple precision variables, and some other routines to allow the user to inspect and modify the base, number of digits, sign, exponent, and digits of multiple precision numbers without needing to know the details of the implementation of the package. (These should be modified only with extreme care, however.) Finally, we added some input/output routines which are simpler to use with the AUGMENT interface than those originally included in the multiple precision arithmetic package. All of these routines were extremely straightforward to write and required a total of about 120 executable statements. A listing of them is given in Appendix C.

In order to interface the PACK and UNPK routines, we introduced another data type called MULTIPAK; the PACK and UNPK routines were then described as conversions between types MULTIPLE and MULTIPAK.

The entire interface was written in less than a half-day; the most time-consuming task was revising the documentation for the multiple precision package!

3. Use of the package via AUGMENT:

As explained in [4], the use of a nonstandard arithmetic package via AUGMENT is extremely simple. The majority of the package modules are invoked automatically by AUGMENT, the exceptions being mainly the input/output and error handling routines.

To use the package through AUGMENT, the user declares all multiple precision variables using statements of the form

```
MULTIPLE X, Y(10), Z
  or
IMPLICIT MULTIPLE (A - H, O - Z)
```

(AUGMENT accepts type declarations via IMPLICIT statements, whether or not the FORTRAN compiler does; this is convenient when converting a program to multiple precision.) The majority of the program is then written just as though MULTIPLE were a standard FORTRAN data type.

If it is desired to store multiple precision variables in packed form, one would declare a 10 by 100 array of packed variables in the following way:

```
MULTIPAK A(10,100)
```

Since the package normally operates only on unpacked variables, any packed variables must normally be converted to unpacked format before use. This may be accomplished by either of two methods:

```
X = A(I, J)        (normal replacement statement)
CTM(A(I, J))       (conversion function)
```

Packed variables should not normally be used directly in arithmetic expressions, since AUGMENT will not generate the appropriate conversion in all cases. Packed variables may be used in certain expressions; for example, if A and C are type MULTIPLE and B is type MULTIPAK, the expression

```
A = B * C
```

will work properly. However, the expression

```
A = EXP(B)
```

will not work; it must be written as

```
A = EXP(CTM(B)).
```

The user may elect to try mixed mode expressions of other kinds; the worst that can happen is that the linkage editor will discover that AUGMENT has generated a call on a nonexistent routine.

Constants may be introduced into the program by statements of the following types:

```
PI = 'PI'
X = '.1$'
```

The dollar sign on the second Hollerith literal is a sentinel to let the Hollerith-unpacking routine know when it has reached the end of the literal. If the compiler generates a sentinel, and if the unpacking routine recognizes it, the terminal '$' is unnecessary. (Note that the Hollerith-unpacking routine is NOT portable; it will need to be rewritten for each new system. The ones shown in Appendix C are for UNIVAC 1100 FORTRAN V, UNIVAC 1100 ASCII FORTRAN, and IBM 360 FORTRAN G or H, respectively.)

The user must still set the various parameters for the package, as explained in [1] and [3]. Care must be exercised to ensure that the dimensions of the multiple precision variables communicated to the package are no greater than those used by AUGMENT in assigning space to the variables. One method of setting the parameters is by including the following statements in the main program, before any (other) executable statements:

```
COMMON IDUMMY(K) (where K = MAXR + 5)
CALL MPSET (LUN, NDIGIT, N, MAXR)
```

where LUN is the logical unit number for output (usually 6); NDIGIT is the number of decimal digits of precision desired; N is the number of storage locations required for each multiple precision variable (this must not exceed the number given in Line 23 of the description deck -- 12 in the deck shown in Appendix B); and MAXR is the length of the working space array as described in [1]. Of course, the user may also set these parameters directly as described in [3], but in that case, care must be exercised not to exceed the number of locations assigned to variables by AUGMENT.

Another way of setting these parameters to default values is to include the statement

```
INITIALIZE MP
```

in the type declarations. This causes AUGMENT to generate a call on the routine MPINIT, which then sets the parameters to values fixed in the MPINIT subroutine. Of course, changes in the dimensions in the description deck must be accompanied by appropriate changes in the parameters in MPINIT if this method is to be used. This is a bit of a cludge, but it works, provided the default values are what one really wants.

A third way of providing these parameters to the package would be even easier, but would require some modification of the package. If all occurrences of blank COMMON were changed to labeled COMMON (e.g., COMMON/MPCOM/), the package parameters could then be set via a DATA statement. (This was not done in the existing package because of a restriction in the ANSI (1966) standards; according to these standards, labeled COMMON must be declared consistently in all routines.) The setting of these parameters in this manner would obviate the need for the user to take any action at all; however, it would result in incompatibility with the standard (published) version of the multiple precision arithmetic package.

The maximum precision available to the user via the given description deck depends on the word length of the host computer; on the UNIVAC 1110, it is approximately 43 digits. The value of MAXR likewise depends on the characteristics of the host machine (and on the modules of the package being used in the program); we used 296 for the UNIVAC 1110 assuming 12 words per multiple precision variable.

If the precision provided by the description deck is not sufficient for the user's needs, it is not a difficult task to increase it; one merely increases the value of N given in Line 23 of the description deck to accommodate the desired precision; increases the value in Line 20 of the description deck to INT((N + 1)/2); and increases MAXR as appropriate. The number of locations needed for the work space array will depend on which of the package routines are being used; the amount of work space needed for each routine is given in [3]. The most space-consuming routines are MPBESJ and MPLNGM. If one wishes to avoid the pain of calculating the precise requirements, one may be assured that by using

```
MAXR = max(T**2 + 15*T + 27, 14*T + 156)
```

where T = N - 2, enough work space will be reserved for any routine in the package. These considerations are discussed in greater detail in [3].

Once the program has been written, the following runstream will invoke AUGMENT and cause the translated program to be written on logical unit 20:

```
(invoke AUGMENT)
(Description Deck)
*BEGIN
(Source Program)
*END
```

The resulting program on logical unit 20 would then be compiled just like any other FORTRAN program; the compiled program would then be linked with the multiple precision library routines and executed.

A complete list of the operations and functions available in the multiple precision arithmetic package, together with the manner in which they are invoked via AUGMENT, is shown in Appendix A.

## 4. Conclusion:

We have demonstrated the method of interfacing a supporting package with the AUGMENT precompiler in the most convincing way possible: by actually doing it.

The interface shown in this paper is self-contained, and can be used (with appropriate modifications, as indicated in the text) with Brent's multiple precision arithmetic package, assuming AUGMENT is available. A revised version of the multiple precision arithmetic package, incorporating the AUGMENT interface routines, is available from the first author.

Questions may be addressed to the authors.

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1. Brent, Richard P., A FORTRAN multiple-precision arithmetic package, Assoc. Comput. Mach. Trans. Math. Software 4 (1978), 57-70.

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## APPENDIX A
## OPERATIONS IMPLEMENTED IN BRENT'S MULTIPLE PRECISION PACKAGE

| OPERATION | DEFINITION/EXPLANATION | RESULT TYPE | ROUTINE INVOCATION VIA AUGMENT | ROUTINE INVOCATION DIRECT | ROUTINE TYPE (AUGMENT) |
|---|---|---|---|---|---|
| ARITHMETIC | | | | | |
| ADDITION | | | | | |
| | Sum of two MP numbers | M | MA + MB | MPADD(MA, MB, MR) | S |
| | Sum of MP number and an integer | M | MA + IB | MPADDI(MA, IB, MR) | S |
| | Sum of MP number and the rational number IB/IC | M | ADDQ(MA, IB, IC) | MPADDQ(MA, IB, IC, MR) | S |
| DIVISION | | | | | |
| | Quotient of two MP numbers | M | MA / MB | MPDIV(MA, MB, MR) | S |
| | Quotient of MP number and an integer | M | MA / IB | MPDIVI(MA, IB, MR) | S |
| MULTIPLICATION | | | | | |
| | Product of two MP numbers | M | MA * MB | MPMUL(MA, MB, MR) | S |
| | Product of MP number and an integer | M | MA * IB | MPMULI(MA, IB, MR) | S |
| | Product of MP number and rat. number IB/IC | M | MULQ(MA, IB, IC) | MPMULQ(MA, IB, IC, MR) | S |
| RECIPROCAL | | | | | |
| | Reciprocal of MP number | M | REC(MA) | MPREC(MA, MR) | S |
| SUBTRACTION | | | | | |
| | Difference of two MP numbers | M | MA - MB | MPSUB(MA, MB, MR) | S |
| POWERS AND ROOTS | | | | | |
| | Raise MP number to integer power | M | MA ** IB | MPPWR(MA, IB, MR) | S |
| | Raise MP number to MP power | M | MA ** MB | MPPWR2(MA, MB, MR) | S |
| | Raise rat. number IA/IB to rat. power IC/ID | M | QPWR(IA,IB,IC,ID) | MPQPWR(IA, IB, IC, ID, MR) | S |
| | IBth root of MP number | M | ROOT(MA, IB) | MPROOT(MA, IB, MR) | S |
| ELEMENTARY FUNCTIONS | | | | | |
| | Absolute value | M | ABS(M) | MPABS(MA, MR) | S |
| | Arc sine of MP number | M | ASIN(MA) | MPASIN(MA, MR) | S |
| | Arc tangent of MP number | M | ATAN(MA) | MPATAN(MA,MR) | S |
| | Arc tangent of 1/IA | M | ART1(IA) | MPART1(IA, MR) | S |
| | Cosine of MP number | M | COS(MA) | MPCOS(MA, MR) | S |
| | Hyperbolic cosine of MP number | M | COSH(MA) | MPCOSH(MA, MR) | S |
| | Exp of MP number | M | EXP(MA) | MPEXP(MA, MR) | S |
| | (Exp - 1) of MP number | M | EXP1(MA) | MPEXP1(MA, MR) | S |
| | Fractional part of MP number | M | FRAC(MA) | MPCMF(MA, MR) | S |
| | Integer part of MP number | M | INT(MA) | MPCMIM(MA, MR) | S |
| | Natural logarithm of MP number | M | LOG(MA) | MPLN(MA, MR) | S |
| | Natural logarithm of MP number using Gauss-Salamin algorithm | M | LNGS(MA) | MPLNGS(MA, MR) | S |
| | Natural logarithm of (1 + MP number) | M | LNS(MA) | MPLNS(MA) | S |
| | Natural logarithm of small positive integer | M | LOG(IA) | MPLNI(IA, MR) | S |
| | Maximum of two MP numbers | M | MAX(MA, MB) | MPMAX(MA, MB, MR) | S |
| | Minimum of two MP numbers | M | MIN(MA, MB) | MPMIN(MA, MB, MR) | S |
| | Sine of MP number | M | SIN(MA) | MPSIN(MA, MR) | S |
| | Hyperbolic sine of MP number | M | SINH(MA) | MPSINH(MA, MR) | S |
| | Square root of MP number | M | SQRT(MA) | MPSQRT(MA, MR) | S |
| | Tangent of MP number | M | TAN(MA) | MPTAN(MA, MR) | S |
| | Hyperbolic tangent of MP number | M | TANH(MA) | MPTANH(MA, MR) | S |

## APPENDIX A (Continued)
## OPERATIONS IMPLEMENTED IN BRENT'S MULTIPLE PRECISION PACKAGE

| OPERATION | DEFINITION/EXPLANATION | RESULT TYPE | ROUTINE INVOCATION VIA AUGMENT | ROUTINE INVOCATION DIRECT | ROUTINE TYPE (AUGMENT) |
|---|---|---|---|---|---|
| SPECIAL FUNCTIONS | | | | | |
| | Bessel function (first kind) of integer order | M | BESJ(MA, IB) | MPBESJ(MA, IB, MR) | S |
| | Dawson's integral of MP argument | M | DAW(MA) | MPDAW(MA, MR) | S |
| | Exponential integral of MP argument | M | EI(MA) | MPEI(MA, MR) | S |
| | Error function of MP number | M | ERF(MA) | MPERF(MA, MR) | S |
| | Complementary error function of MP number | M | ERFC(MA) | MPERFC(MA, MR) | S |
| | Gamma function of MP number | M | GAM(MA) | MPGAM(MA, MR) | S |
| | Gamma function of rational number IA/IB | M | GAM(IA, IB) | MPGAMQ(IA, IB, MR) | S |
| | GCD of two MP integers | M | GCD(MA, MB) | MPGCDA(MA, MB, MR) | S |
| | Logarithmic integral of MP number | M | LI(MA) | MPLI(MA, MR) | S |
| | Logarithm of gamma function of MP number | M | LNGM(MA) | MPLNGM(MA, MR) | S |
| | Riemann zeta function of pos. integer | M | ZETA(IA) | MPZETA(IA, MR) | S |
| CONSTANTS | | | | | |
| | Bernoulli numbers | PM | BERN(IA, IB) | MPBERN(IA, IB, PMR) | S |
| | Multiple precision machine precision | M | CTM('EPS') | MPEPS(MR) | S |
| | Euler's constant | M | CTM('EUL') | MPEUL(MR) | S |
| | Largest positive MP number | M | CTM('MAXR') | MPMAXR(MR) | S |
| | Smallest positive MP number | M | CTM('MINR') | MPMINR(MR) | S |
| | Pi | M | CTM('PI') | MPPI(MR) | S |
| INPUT/OUTPUT | | | | | |
| | Read IB words from Unit IC, under Format HD, convert to MP number MR; LR is error code. | L | MPINF(MR,IB,IC,HD) | MPINF(MR,IB,IC,HD,LR) | S |
| | Write IB words, representing MP number MA, on Unit LUN, IC places after decimal point, under format HD; LR is error code. | L | MPOUTF(MA,IB,IC,HD) | MPOUTF(MA,IB,IC,HD,LR) | S |
| | Dump MP number on Logical Unit LUN | - | - | MPDUMP(MA) | S |
| | Convert unpacked Hollerith fixed point to MP | M | - | MPIN(UHA, MR, IB, IR) | S |
| | Convert upkd Hol fixed pt. + exp IC to MP | M | - | MPINE(UHA,MR,IB,IC,IR) | S |
| | Convert MP to upkd Hol (fixed pt.) | UH | - | MPOUT(MA, UHR, IC, ID) | S |
| | Convert MP to upkd Hol. (floating pt.) | UH, I | - | MPOUTE(MA, UHR, IR, ID) | S |
| CONVERSION | | | | | |
| | Double Precision to Multiple | M | CTM(DA) | MPCDM(DA, MR) | S |
| | Integer to Multiple | M | CTM(IA) | MPCIM(IA, MR) | S |
| | Real to Multiple | M | CTM(RA) | MPCRM(RA, MR) | S |
| | Rational IA/IB to Multiple | M | CTM(IA, IB) | MPCQM(IA, IB, MR) | S |
| | Packed Multiple to Multiple | M | CTM(PMA) | MPUNPK(PMA,MR) | S |
| | Packed Hollerith to Multiple | M | CTM(HA) | MPCAM(HA, MR) | S |
| | Multiple to Double Precision | D | CTD(MA) | MPCMD(MA, DR) | S |
| | Multiple to Integer | I | CTI(MA) | MPCMI(MA, IR) | S |
| | Multiple to Real | R | CTR(MA) | MPCMR(MA, IR) | S |
| | Multiple to Packed Multiple | PM | CTP(MA) | MPPACK(MA, PR) | S |
| | Multiple to Double Precision + Integer exponent | D, I | - | MPCMDE(MA, IR, DR) | S |
| | Multiple to Multiple + Integer exponent | M, I | - | MPCMEF(MA, IR, MR) | S |
| | Multiple to Real + Integer exponent | M, I | - | MPCMRE(MA, IR, RR) | S |

APPENDIX A (Continued)

OPERATIONS IMPLEMENTED IN BRENT'S MULTIPLE PRECISION PACKAGE

| OPERATION | DEFINITION/EXPLANATION | RESULT TYPE | ROUTINE INVOCATION VIA AUGMENT | ROUTINE INVOCATION DIRECT | ROUTINE TYPE (AUGMENT) |
|---|---|---|---|---|---|
| COMPARISON | ( +1 if >, 0 if =, -1 if <) | | | | |
| | Compare absolute value of MP numbers | I | CMPA(MA, MB) | MPCMPA(MA, MB) | I |
| | Compare MP number with integer | I | COMP(MA, IB) | MPCMPI(MA, IB) | I |
| | Compare MP number with real | I | COMP(MA, RB) | MPCMPR(MA, RB) | I |
| | Compare MP numbers | I | COMP(MA, MB) | MPCOMP(MA, MB) | I |
| RELATIONAL | | | | | |
| | MA equal to MB | L | MA .EQ. MB | MPEQ(MA, MB) | L |
| | MA greater than or equal to MB | L | MA .GE. MB | MPGE(MA, MB) | L |
| | MA greater than MB | L | MA .GT. MB | MPGT(MA, MB) | L |
| | MA less than or equal to MB | L | MA .LE. MB | MPLE(MA, MB) | L |
| | MA less than MB | L | MA .LT. MB | MPLT(MA, MB) | L |
| | MA not equal to MB | L | MA .NE. MB | MPNE(MA, MB) | L |
| TEST | | | | | |
| | Three-way branch | - | IF(MA)N1,N2,N3 | IF(MA(1))N1,N2,N3 | - |
| FIELD FUNCTIONS | | | | | |
| | Sign of MP number | I | SGN(MA) | MPSIGB(IA,MR)(insertion) | S |
| | | | | MPSIGA(MA)(extraction) | I |
| | Exponent of MP number | I | EXPON(MA) | MPEXPB(IA, MR)(insertion) | S |
| | | | | MPEXPA(MA)(extraction) | I |
| | IBth digit of MP number | I | DIGIT(MA, IB) | MPDGB(IA, MR, IB)(insertion) | S |
| | | | | MPDGA(MA, IB)(extraction) | I |
| | Number of MP digits | I | NUMDIG(MDUMMY) | MPDIGB(IA, DUMMY)(insertion) | S |
| | | | | MPDIGA(DUMMY)(extraction) | I |
| | Maximum exponent of MP number | I | MAXEXP(MDUMMY) | MPMEXB(IA, DUMMY)(insertion) | S |
| | | | | MPMEXA(DUMMY)(extraction) | I |
| | MP base | I | BASE(MDUMMY) | MPBASB(IA, DUMMY)(insertion) | S |
| | | | | MPBASA(DUMMY)(extraction) | I |
| UTILITY | | | | | |
| | Unary minus | M | -MA | MPNEG(MA, MR) | S |
| | Replacement | M | MR = MA | MPSTR(MA, MR) | S |
| | | PM | PMR = PMA | MPKSTR(PMA, PMR) | S |
| | Evaluate Polynomial (integer coefs, dim IC) | M | - | MPPOLY(MA, MR, IB, IC) | S |
| | Set parameters for MP routines | - | - | MPSET(IA, IB, IC, ID) | S |
| | Clear next IB positions of MP number | - | - | MPCLR(MR, IB) | S |
| ERROR DETECTION | | | | | |
| | Check legality of parameters to MP package | - | - | MPCHK(IA, IB) | S |
| | Handle fatal error conditions | - | - | MPERR | S |
| | Handle MP overflow | - | - | MPOVFL(MR) | S |
| | Handle MP underflow | - | - | MPUNFL(MR) | S |

## APPENDIX A (Continued)
## OPERATIONS IMPLEMENTED IN BRENT'S MULTIPLE PRECISION PACKAGE

NOTES ON TABLE:

1. Data types are indicated by one- or two-letter abbreviations: D = DOUBLE PRECISION; H = PACKED HOLLERITH; I = INTEGER; L = LOGICAL; M = MULTIPLE; PM = PACKED MULTIPLE; R = REAL; UH = UNPACKED HOLLERITH.

2. Variable names: The first letter (or pair of letters) indicates the data type of the variable as above. The terminal letter is A, B, C, or D for an argument; R for result.

3. Routine types: S denotes subroutine; any other letter denotes a function of the designated type.

4. A field function is one which allows specified portions of a data element to be altered or retrieved selectively. Extreme care should be used in altering fields of data elements.

5. The conversion routines (those beginning with 'CT' in Column 4) may also be invoked implicitly via replacement statements. For example, the statement "MR = DA" will cause AUGMENT to generate a call on MPCDM as shown in Column 5.

6. In some cases, AUGMENT will recognize synonyms of the names given in Column 4. Particulars may be found in the description deck (Appendix B). Of course, the user may change or add to the recognition names by modifying the description deck; see [5] for details.

## APPENDIX B

### AUGMENT DESCRIPTION DECK FOR BRENT'S MULTIPLE PRECISION ARITHMETIC PACKAGE

```
*DESCRIBE MULTIPAK
COMMENT  AUGMENT DESCRIPTION DECK FOR THE MULTIPLE-PRECISION
         ARITHMETIC PACKAGE OF R. P. BRENT, UNIVAC 1100 VERSION.
         THREE TYPES OF VARIABLE ARE DEFINED HERE -
               MULTIPLE   (STANDARD MULTIPLE-PRECISION NUMBERS),
               MULTIPAK   (PACKED MULTIPLE-PRECISION NUMBERS), AND
               INITIALIZE (USED ONLY AS A DEVICE TO PERSUADE
                           AUGMENT TO INITIALIZE THE MP PACKAGE).
         WORKING SPACE SHOULD BE ALLOCATED AND THE MP PACKAGE
         INITIALIZED BY THE DECLARATION
               INITIALIZE MP
         IN THE MAIN PROGRAM.
         THIS DESCRIPTION DECK ASSUMES THAT MULTIPLE PRECISION NUMBERS
         WILL HAVE NO MORE THAN 10 DIGITS (BASE 65536) FOR A TOTAL
         PRECISION NOT EXCEEDING ABOUT 43 DECIMAL PLACES.  FOR THIS,
         EACH MP NUMBER REQUIRES 12 WORDS (6 IN PACKED FORMAT).
         SEE COMMENTS IN ROUTINE MPINIT FOR THE METHOD OF CHANGING
         THE PRECISION OR ADAPTING TO A MACHINE WITH WORDLENGTH
         OTHER THAN 36 BITS.
DECLARE INTEGER(6), KIND SAFE SUBROUTINE, PREFIX MPK
SERVICE COPY(STR)
*DESCRIBE MULTIPLE
DECLARE INTEGER(12), KIND SAFE SUBROUTINE, PREFIX MP
OPERATOR + (,NULL UNARY, PRV, $), - (NEG, UNARY),
         + (ADD, BINARY3, PRV, $, $, $, COMM), * (MUL),
         - (SUB,,,,,, NONCOMM), / (DIV), ** (PWR2),
         + (ADDI,,,, INTEGER), * (MULI), / (DIVI), ** (PWR),
         .EQ. (EQ, BINARY2, PRV, $, LOGICAL, COMM), .NE. (NE),
         .GE. (GE,,,,, NONCOMM), .GT. (GT), .LE. (LE), .LT. (LT)
TEST     MPSIGA (SIGA, INTEGER)
FIELD    SGN (SIGA, SIGB, ($), INTEGER),
         EXPON (EXPA, EXPB), BASE (BASA, BASB), NUMDIG (DIGA, DIGB),
         MAXEXP (MEXA, MEXB), DIGIT (DGA, DGB, ($, INTEGER))
FUNCTION ABS (ABS, ($), $), ASIN (ASIN), ATAN (ATAN), CMF (CMF),
         CMIM (CMIM), COS (COS), COSH (COSH), DAW (DAW), EI (EI),
         ERF (ERF), ERFC (ERFC), EXP (EXP), EXP1 (EXP1), FRAC (CMF),
         GAM (GAM), INT (CMIM), LI (LI), LN (LN), LOG (LN), LNGM (LNGM),
         LNGS (LNGS), LNS (LNS), REC (REC), SIN (SIN), SINH (SINH),
         SQRT (SQRT), TAN (TAN), TANH (TANH),
         ART1 (ART1, (INTEGER)), LN (LNI), LNI (LNI), LOG (LNI),
         ZETA (ZETA), CAM (CAM), CAM (CAM, (HOLLERITH)),
         MAX (MAX, ($, $)), MIN (MIN), GCD (GCDA),
         BESJ (BESJ, ($, INTEGER)), ROOT (ROOT),
         MPINF (INF(SUBROUTINE),($,INTEGER,INTEGER,HOLLERITH),LOGICAL),
         MPOUTF (OUTF(SUBROUTINE)),
         MPINF (INF(SUBROUTINE), ($, INTEGER, INTEGER, INTEGER)),
         MPOUTF (OUTF(SUBROUTINE)),
         COMP (COMP, ($, $), INTEGER), CMPA (CMPA),
         COMP (CMPI, ($, INTEGER)), COMP (CMPR, ($, REAL)),
         ADDQ (ADDQ, ($, INTEGER, INTEGER), $), MULQ (MULQ),
```

```
        QPWR (QPWR, (INTEGER, INTEGER, INTEGER, INTEGER)),
        CQM (CQM, (INTEGER, INTEGER)), CTM (CQM),
        GAM (GAMQ), GAMQ (GAMQ),
        BERN (BERN, (INTEGER, INTEGER), MULTIPAK)
CONVERSION CTM (CDM, DOUBLE PRECISION, $, UPWARD),
           CTM (CIM, INTEGER), CTM (CRM, REAL),
           CTM (UNPK, MULTIPAK), CTM (CAM, HOLLERITH),
           CTD (CMD(SUBROUTINE), $, DOUBLE PRECISION, DOWNWARD),
           CTI (CMI(SUBROUTINE),, INTEGER),
           CTR (CMR(SUBROUTINE),, REAL), CTP (PACK,, MULTIPAK)
SERVICE COPY (STR)
*DESCRIBE INITIALIZE
DECLARE INTEGER(1), KIND SAFE SUBROUTINE, PREFIX MPI
SERVICE COPY (STR), INITIAL (NIT)
COMMENT    END OF AUGMENT DESCRIPTION DECK FOR MP PACKAGE
```

## APPENDIX C

## AUGMENT INTERFACE ROUTINES FOR BRENT'S MULTIPLE PRECISION ARITHMETIC PACKAGE

```
C $$                         ******  MPBASA  ******                MP009551
      FUNCTION MPBASA (X)                                          MP009553
C RETURNS THE MP BASE (FIRST WORD IN COMMON).                      MP009555
C X IS A DUMMY MP ARGUMENT.                                        MP009557
      COMMON B, T, M, LUN, MXR, R                                  MP009559
      INTEGER B, T, R(1), X(1)                                     MP009561
      MPBASA = B                                                   MP009563
      RETURN                                                       MP009565
      END                                                          MP009567

C $$                         ******  MPBASB  ******                MP009571
      SUBROUTINE MPBASB (I, X)                                     MP009573
C SETS THE MP BASE (FIRST WORD OF COMMON) TO I.                    MP009575
C I SHOULD BE AN INTEGER SUCH THAT I .GE. 2                        MP009577
C AND (8*I*I-1) IS REPRESENTABLE AS A SINGLE-PRECISION INTEGER.    MP009579
C X IS A DUMMY MP ARGUMENT (AUGMENT EXPECTS ONE).                  MP009581
      COMMON B, T, M, LUN, MXR, R                                  MP009583
      INTEGER B, T, R(1), X(1)                                     MP009585
C SET BASE TO I, THEN CHECK VALIDITY                               MP009587
      B = I                                                        MP009589
      CALL MPCHK (1, 4)                                            MP009591
      RETURN                                                       MP009593
      END                                                          MP009595

C $$                         ******  MPCAM  ******                 MP012491
      SUBROUTINE MPCAM (A, X)                                      MP012493
C CONVERTS THE HOLLERITH STRING A TO AN MP NUMBER X.               MP012495
C A CAN BE A STRING OF DIGITS ACCEPTABLE TO ROUTINE MPIN           MP012497
C AND TERMINATED BY A DOLLAR ($), E.G. 7H-5.367$,                  MP012499
C OR ONE OF THE FOLLOWING SPECIAL STRINGS -                        MP012501
C            EPS  (MP MACHINE-PRECISION, SEE MPEPS),               MP012503
C            EUL  (EULERS CONSTANT 0.5772..., SEE MPEUL),          MP012505
C            MAXR (LARGEST VALID MP NUMBER, SEE MPMAXR),           MP012507
C            MINR (SMALLEST POSTIVE MP NUMBER, SEE MPMINR),        MP012509
C            PI   (PI = 3.14..., SEE MPPI).                        MP012511
C ONLY THE FIRST TWO CHARACTERS OF THESE STRINGS ARE CHECKED.      MP012513
C SPACE REQUIRED IS NO MORE THAN 5*T+L+14, WHERE L IS THE          MP012515
C NUMBER OF CHARACTERS IN THE STRING A (EXCLUDING $).              MP012517
C IF SPACE IS LESS 3*T+L+11 THE STRING A WILL EFFECTIVELY BE TRUNCATED MP012519
      COMMON B, T, M, LUN, MXR, R                                  MP012521
      INTEGER B, T, R(1), A(1), X(1), ERROR, C(6), D(2)            MP012523
      DATA C(1) /1HA/, C(2) /1HE/, C(3) /1HI/                      MP012525
      DATA C(4) /1HM/, C(5) /1HP/, C(6) /1HU/                      MP012527
C UNPACK FIRST 2 CHARACTERS OF A                                   MP012529
      CALL MPUPK (A, D, 2, N)                                      MP012531
      IF (N.NE.2) GO TO 10                                         MP012533
C SET X TO ZERO AFTER SAVING A(1) IN CASE A AND X COINCIDE         MP012535
      I = A(1)                                                     MP012537
      X(1) = 0                                                     MP012539
C CHECK FOR SPECIAL STRINGS                                        MP012541
```

```
      IF ((D(1).EQ.C(2)).AND.(D(2).EQ.C(5))) CALL MPEPS (X)            MP012543
      IF ((D(1).EQ.C(2)).AND.(D(2).EQ.C(6))) CALL MPEUL (X)            MP012545
      IF ((D(1).EQ.C(4)).AND.(D(2).EQ.C(1))) CALL MPMAXR (X)           MP012547
      IF ((D(1).EQ.C(4)).AND.(D(2).EQ.C(3))) CALL MPMINR (X)           MP012549
      IF ((D(1).EQ.C(5)).AND.(D(2).EQ.C(3))) CALL MPPI (X)             MP012551
C RETURN IF X NONZERO (SO ONE OF ABOVE TESTS SUCCEEDED)                MP012553
      IF (X(1).NE.0) RETURN                                            MP012555
C RESTORE A(1) AND UNPACK, THEN CALL MPIN TO DECODE.                   MP012557
      A(1) = I                                                         MP012559
   10 I2 = 3*T + 12                                                    MP012561
      CALL MPUPK (A, R(I2), MXR+1-I2, N)                               MP012563
      CALL MPIN (R(I2), X, N, ERROR)                                   MP012565
      IF (ERROR.EQ.0) RETURN                                           MP012567
      WRITE (LUN, 20)                                                  MP012569
   20 FORMAT (53H *** ERROR IN HOLLERITH CONSTANT IN CALL TO MPCAM ***) MP012571
      CALL MPERR                                                       MP012573
      RETURN                                                           MP012575
      END                                                              MP012577

C $$                   ******   MPDGA   ******                         MP019741
      FUNCTION MPDGA (X, N)                                            MP019743
C RETURNS THE N-TH DIGIT OF THE MP NUMBER X FOR 1 .LE. N .LE. T.       MP019745
C RETURNS ZERO IF X IS ZERO OR N .LE. 0 OR N .GT. T.                   MP019747
      COMMON B, T, M, LUN, MXR, R                                      MP019749
      INTEGER B, T, R(1), X(1)                                         MP019751
      MPDGA = 0                                                        MP019753
      IF ((X(1).NE.0).AND.(N.GT.0).AND.(N.LE.T)) MPDGA = X(N+2)        MP019755
      RETURN                                                           MP019757
      END                                                              MP019759

C $$                   ******   MPDGB   ******                         MP019781
      SUBROUTINE MPDGB (I, X, N)                                       MP019783
C SETS THE N-TH DIGIT OF THE MP NUMBER X TO I.                         MP019785
C N MUST BE IN THE RANGE 1 .LE. N .LE T,                               MP019787
C I MUST BE IN THE RANGE 0 .LE. I .LT. B                               MP019789
C (AND I .NE. 0 IF N .EQ. 1).                                          MP019791
C THE SIGN AND EXPONENT OF X ARE UNCHANGED.                            MP019793
      COMMON B, T, M, LUN, MXR, R                                      MP019795
      INTEGER B, T, R(1), X(1)                                         MP019797
      IF ((N.GT.0).AND.(N.LE.T)) GO TO 20                              MP019799
      WRITE (LUN, 10)                                                  MP019801
   10 FORMAT (48H *** DIGIT POSITION ILLEGAL IN CALL TO MPDGB ***)     MP019803
      GO TO 40                                                         MP019805
   20 IF ((I.GE.0).AND.(I.LT.B).AND.((I+N).GT.1)) GO TO 50             MP019807
      WRITE (LUN, 30)                                                  MP019809
   30 FORMAT (45H *** DIGIT VALUE ILLEGAL IN CALL TO MPDGB ***)        MP019811
   40 CALL MPERR                                                       MP019813
      RETURN                                                           MP019815
   50 X(N+2) = I                                                       MP019817
      RETURN                                                           MP019819
      END                                                              MP019821

C $$                   ******   MPDIGA   ******                        MP019841
      FUNCTION MPDIGA (X)                                              MP019843
C RETURNS THE NUMBER OF MP DIGITS (SECOND WORD IN COMMON).             MP019845
C X IS A DUMMY MP ARGUMENT.                                            MP019847
```

```
      COMMON B, T, M, LUN, MXR, R                                       MP019849
      INTEGER B, T, R(1), X(1)                                          MP019851
      MPDIGA = T                                                        MP019853
      RETURN                                                            MP019855
      END                                                               MP019857

C $$                   ******  MPDIGB  ******                           MP019861
      SUBROUTINE MPDIGB (I, X)                                          MP019863
C SETS THE NUMBER OF MP DIGITS (SECOND WORD OF COMMON) TO I.            MP019865
C I SHOULD BE AN INTEGER SUCH THAT I .GE. 2                             MP019867
C X IS A DUMMY MP ARGUMENT (AUGMENT EXPECTS ONE).                       MP019869
C WARNING *** MP NUMBERS MUST BE DECLARED AS INTEGER ARRAYS OF          MP019871
C         *** DIMENSION AT LEAST I+2. MPDIGB DOES NOT CHECK THIS.        MP019873
      COMMON B, T, M, LUN, MXR, R                                       MP019875
      INTEGER B, T, R(1), X(1)                                          MP019877
C SET DIGITS TO I, THEN CHECK VALIDITY                                  MP019879
      T = I                                                             MP019881
      CALL MPCHK (1, 4)                                                 MP019883
      RETURN                                                            MP019885
      END                                                               MP019887

C $$                   ******  MPEQ  ******                             MP023221
      LOGICAL FUNCTION MPEQ (X, Y)                                      MP023223
C RETURNS LOGICAL VALUE OF (X .EQ. Y) FOR MP X AND Y.                   MP023225
      INTEGER X(1), Y(1)                                                MP023227
      MPEQ = (MPCOMP(X,Y) .EQ. 0)                                       MP023229
      RETURN                                                            MP023231
      END                                                               MP023233

C $$                   ******  MPEXPA  ******                           MP027271
      FUNCTION MPEXPA (X)                                               MP027273
C RETURNS THE EXPONENT OF THE MP NUMBER X                               MP027275
C (OR LARGEST NEGATIVE EXPONENT IF X IS ZERO).                          MP027277
      COMMON B, T, M, LUN, MXR, R                                       MP027279
      INTEGER B, T, R(1), X(2)                                          MP027281
      MPEXPA = -M                                                       MP027283
C RETURN -M IF X ZERO, X(2) OTHERWISE                                   MP027285
      IF (X(1).NE.0) MPEXPA = X(2)                                      MP027287
      RETURN                                                            MP027289
      END                                                               MP027291

C $$                   ******  MPEXPB  ******                           MP027311
      SUBROUTINE MPEXPB (I, X)                                          MP027313
C SETS EXPONENT OF MP NUMBER X TO I UNLESS X IS ZERO                    MP027315
C (WHEN EXPONENT IS UNCHANGED).                                         MP027317
C X MUST BE A VALID MP NUMBER (EITHER ZERO OR NORMALIZED).              MP027319
      COMMON B, T, M, LUN, MXR, R                                       MP027321
      INTEGER B, T, R(1), X(3)                                          MP027323
C RETURN IF X IS ZERO                                                   MP027325
      IF (X(1).EQ.0) RETURN                                             MP027327
C CHECK FOR VALID MP SIGN AND LEADING DIGIT                             MP027329
      IF ((IABS(X(1)).LE.1).AND.(X(3).GT.0).AND.(X(3).LT.B))            MP027331
     $    GO TO 20                                                      MP027333
      WRITE (LUN, 10)                                                   MP027335
   10 FORMAT (48H *** X NOT VALID MP NUMBER IN CALL TO MPEXPB ***)      MP027337
      CALL MPERR                                                        MP027339
```

```
      X(1) = 0                                                          MP027341
      RETURN                                                            MP027343
C SET EXPONENT OF X TO I                                                MP027345
   20 X(2) = I                                                          MP027347
C CHECK FOR OVERFLOW AND UNDERFLOW                                      MP027349
      IF (I.GT.M) CALL MPOVFL (X)                                       MP027351
      IF (I.LT.(-M)) CALL MPUNFL (X)                                    MP027353
      RETURN                                                            MP027355
      END                                                               MP027357

C $$                   ******  MPGE  ******                             MP030521
      LOGICAL FUNCTION MPGE (X, Y)                                      MP030523
C RETURNS LOGICAL VALUE OF (X .GE. Y) FOR MP X AND Y.                   MP030525
      INTEGER X(1), Y(1)                                                MP030527
      MPGE = (MPCOMP(X,Y) .GE. 0)                                       MP030529
      RETURN                                                            MP030531
      END                                                               MP030533

C $$                   ******  MPGT  ******                             MP030541
      LOGICAL FUNCTION MPGT (X, Y)                                      MP030543
C RETURNS LOGICAL VALUE OF (X .GT. Y) FOR MP X AND Y.                   MP030545
      INTEGER X(1), Y(1)                                                MP030547
      MPGT = (MPCOMP(X,Y) .GT. 0)                                       MP030549
      RETURN                                                            MP030551
      END                                                               MP030553

C $$                   ******  MPINF  ******                            MP032761
      SUBROUTINE MPINF (X, N, UNIT, IFORM, ERR)                         MP032763
C READS N WORDS FROM LOGICAL UNIT IABS(UNIT) USING FORMAT IN IFORM,     MP032765
C THEN CONVERTS TO MP NUMBER X USING ROUTINE MPIN.                      MP032767
C IFORM SHOULD CONTAIN A FORMAT WHICH ALLOWS FOR READING N WORDS        MP032769
C IN A1 FORMAT, E.G. 6H(80A1)                                           MP032771
C ERR RETURNED AS TRUE IF MPIN COULD NOT INTERPRET INPUT AS             MP032773
C AN MP NUMBER OR IF N NOT POSITIVE, OTHERWISE FALSE.                   MP032775
C IF ERR IS TRUE THEN X IS RETURNED AS ZERO.                            MP032777
C SPACE REQUIRED 3T+N+11.                                               MP032779
      COMMON B, T, M, LUN, MXR, R                                       MP032781
      INTEGER B, T, R(1), X(1), UNIT, IFORM(1)                          MP032783
      LOGICAL ERR                                                       MP032785
C CHECK THAT ENOUGH SPACE AVAILABLE                                     MP032787
      CALL MPCHK (3, N+11)                                              MP032789
      I2 = 3*T + 12                                                     MP032791
C READ N WORDS UNDER FORMAT IFORM.                                      MP032793
      CALL MPIO (R(I2), N, (-IABS(UNIT)), IFORM, ERR)                   MP032795
      X(1) = 0                                                          MP032797
C RETURN IF ERROR                                                       MP032799
      IF (ERR) RETURN                                                   MP032801
C ELSE CONVERT TO MP NUMBER.                                            MP032803
      CALL MPIN (R(I2), X, N, IER)                                      MP032805
C RETURN ERROR FLAG IF MPIN OBJECTED                                    MP032807
      ERR = (IER.NE.0)                                                  MP032809
      RETURN                                                            MP032811
      END                                                               MP032813

C $$                   ******  MPINIT  ******                           MP032821
      SUBROUTINE MPINIT (X)                                             MP032823
```

```
C DECLARES BLANK COMMON (USED BY MP PACKAGE) AND
C CALLS MPSET TO INITIALIZE PARAMETERS
C THE AUGMENT DECLARATION
C         INITIALIZE MP
C CAUSES A CALL TO MPINIT TO BE GENERATED.
C *** ASSUMES OUTPUT UNIT 6, 43 DECIMAL PLACES,
C *** 10 MP DIGITS, SPACE 296 WORDS.  IF THE AUGMENT
C *** DESCRIPTION DECK IS CHANGED THIS ROUTINE SHOULD
C *** BE CHANGED ACCORDINGLY.
      COMMON B, T, M, LUN, MXR, R
      INTEGER B, T, X(1)
C THE STATEMENTS
      INTEGER R(296)
      CALL MPSET (6, 43, 12, 296)
C ARE A SPECIAL CASE OF
C     INTEGER R(MXR)
C     CALL MPSET (LUN, IDECPL, T+2, MXR)
C WHERE LUN IS THE LOGICAL UNIT FOR OUTPUT,
C IDECPL IS THE EQUIVALENT NUMBER OF DECIMAL PLACES REQUIRED,
C T IS THE NUMBER OF MP DIGITS, AND
C MXR IS THE SIZE OF THE WORKING AREA USED BY MP
C (MXR = MAX (T*T+15*T+27, 14*T+156) IS SUFFICIENT).
C TO CHANGE THE PRECISION, MODIFY THE DIMENSIONS IN THE
C DECLARE STATEMENTS IN THE AUGMENT DESCRIPTION DECK -
C THE DIMENSION FOR TYPE MULTIPLE SHOULD BE T+2 AND
C FOR TYPE MULTIPAK SHOULD BE INT ((T+3)/2).
C SEE COMMENTS IN ROUTINE MPSET FOR THE NUMBER OF MP
C DIGITS REQUIRED TO GIVE THE EQUIVALENT OF ANY DESIRED
C NUMBER OF DECIMAL PLACES.
C *** ON SOME SYSTEMS A DECLARATION OF BLANK COMMON IN THE MAIN
C *** PROGRAM MAY BE NECESSARY.  IF SO, DECLARE
C ***         COMMON MPWORK(301)
C *** OR, MORE GENERALLY,
C ***         COMMON MPWORK(MXR+5)
C *** IN THE MAIN PROGRAM.
      RETURN
      END

C $$                        ******  MPIO  ******
      SUBROUTINE MPIO (C, N, UNIT, IFORM, ERR)
C IF UNIT .GT. 0 WRITES C(1), ... , C(N) IN FORMAT IFORM
C IF UNIT .LE. 0 READS  C(1), ... , C(N) IN FORMAT IFORM
C IN BOTH CASES USES LOGICAL UNIT IABS(UNIT).
C ERR IS RETURNED AS TRUE IF N NON-POSITIVE, OTHERWISE FALSE.
C WE WOULD LIKE TO RETURN ERR AS TRUE IF READ/WRITE ERROR DETECTED,
C BUT THIS CAN NOT BE DONE WITH ANSI STANDARD FORTRAN (1966).
C *** UNIVAC ASCII FORTRAN (FTN 5R1AE) DOES NOT WORK IF IFORM
C *** IS DECLARED WITH DIMENSION 1.  MOST FORTRANS DO THOUGH.
      INTEGER C(N), UNIT, IFORM(20)
      LOGICAL ERR
      ERR = (N.LE.0)
      IF (ERR) RETURN
      IU = IABS(UNIT)
      IF (UNIT.GT.0) WRITE (IU, IFORM) C
      IF (UNIT.LE.0) READ  (IU, IFORM) C
      RETURN
```

```
      END                                                               MP032957

C $$                   ******  MPKSTR  ******                           MP032961
      SUBROUTINE MPKSTR (X, Y)                                          MP032963
C SETS Y = X FOR PACKED MP NUMBERS X AND Y.                             MP032965
C ASSUMES SAME PACKED FORMAT AS MPPACK AND MPUNPK.                      MP032967
      COMMON B, T, M, LUN, MXR, R                                       MP032969
      INTEGER B, T, R(1), X(2), Y(2)                                    MP032972
      Y(2) = X(2)                                                       MP032973
C CHECK FOR ZERO                                                        MP032975
      IF (Y(2).EQ.0) RETURN                                             MP032977
C HERE X NONZERO SO MOVE PACKED NUMBER                                  MP032979
      N = (T+3)/2                                                       MP032981
      DO 10 I = 1, N                                                    MP032983
   10 Y(I) = X(I)                                                       MP032985
      RETURN                                                            MP032987
      END                                                               MP032989

C $$                   ******  MPLE  ******                             MP033001
      LOGICAL FUNCTION MPLE (X, Y)                                      MP033003
C RETURNS LOGICAL VALUE OF (X .LE. Y) FOR MP X AND Y.                   MP033005
      INTEGER X(1), Y(1)                                                MP033007
      MPLE = (MPCOMP(X,Y) .LE. 0)                                       MP033009
      RETURN                                                            MP033011
      END                                                               MP033013

C $$                   ******  MPLT  ******                             MP037281
      LOGICAL FUNCTION MPLT (X, Y)                                      MP037283
C RETURNS LOGICAL VALUE OF (X .LT. Y) FOR MP X AND Y.                   MP037285
      INTEGER X(1), Y(1)                                                MP037287
      MPLT = (MPCOMP(X,Y) .LT. 0)                                       MP037289
      RETURN                                                            MP037291
      END                                                               MP037293

C $$                   ******  MPMEXA  ******                           MP038051
      FUNCTION MPMEXA (X)                                               MP038053
C RETURNS THE MAXIMUM ALLOWABLE EXPONENT OF MP NUMBERS (THE THIRD       MP038055
C WORD OF COMMON).  X IS A DUMMY MP ARGUMENT.                           MP038057
      COMMON B, T, M, LUN, MXR, R                                       MP038059
      INTEGER B, T, R(1), X(1)                                          MP038061
      MPMEXA = M                                                        MP038063
      RETURN                                                            MP038065
      END                                                               MP038067

C $$                   ******  MPMEXB  ******                           MP038071
      SUBROUTINE MPMEXB (I, X)                                          MP038073
C SETS THE MAXIMUM ALLOWABLE EXPONENT OF MP NUMBERS (I.E. THE           MP038075
C THIRD WORD OF COMMON) TO I.                                           MP038077
C I SHOULD BE GREATER THAN T, AND 4*I SHOULD BE REPRESENTABLE           MP038079
C AS A SINGLE-PRECISION INTEGER.                                        MP038081
C X IS A DUMMY MP ARGUMENT (AUGMENT EXPECTS ONE).                       MP038083
      COMMON B, T, M, LUN, MXR, R                                       MP038085
      INTEGER B, T, R(1), X(1)                                          MP038087
      M = I                                                             MP038089
C CHECK LEGALITY OF M.  IF TOO LARGE, 4*M MAY OVERFLOW AND TEST .LE. 0  MP038091
      IF ((M.GT.T).AND.((4*M).GT.0)) RETURN                             MP038093
```

```
      WRITE (LUN, 10)                                                   MP038095
   10 FORMAT (44H *** ATTEMPT TO SET ILLEGAL MAXIMUM EXPONENT,          MP038097
     $        22H IN CALL TO MPMEXB ***)                                MP038099
      CALL MPERR                                                        MP038101
      RETURN                                                            MP038103
      END                                                               MP038105

C $$                   ******  MPNE  ******                             MP040461
      LOGICAL FUNCTION MPNE (X, Y)                                      MP040463
C RETURNS LOGICAL VALUE OF (X .NE. Y) FOR MP X AND Y.                   MP040465
      INTEGER X(1), Y(1)                                                MP040467
      MPNE = (MPCOMP(X,Y) .NE. 0)                                       MP040469
      RETURN                                                            MP040471
      END                                                               MP040473

C $$                   ******  MPOUTF  ******                           MP041781
      SUBROUTINE MPOUTF (X, P, N, IFORM, ERR)                           MP041783
C WRITES MP NUMBER X ON LOGICAL UNIT LUN (FOURTH WORD OF COMMON)        MP041785
C IN FORMAT IFORM AFTER CONVERTING TO FP.N DECIMAL REPRESENTATION       MP041787
C USING ROUTINE MPOUT. FOR FURTHER DETAILS SEE COMMENTS IN MPOUT.       MP041789
C IFORM SHOULD CONTAIN A FORMAT WHICH ALLOWS FOR OUTPUT OF P            MP041791
C WORDS IN A1 FORMAT, PLUS ANY DESIRED HEADINGS, SPACING ETC.           MP041793
C E.G. 24H(8H1HEADING/(11X,100A1))                                      MP041795
C ERR RETURNED AS TRUE IF P NOT POSITIVE, OTHERWISE FALSE.              MP041797
C SPACE REQUIRED 3T+P+11 WORDS.                                         MP041799
      COMMON B, T, M, LUN, MXR, R                                       MP041801
      INTEGER B, T, R(1), X(1), IFORM(1), P                             MP041803
      LOGICAL ERR                                                       MP041805
      ERR = .TRUE.                                                      MP041807
C RETURN WITH ERROR FLAG SET IF OUTPUT FIELD WIDTH P NOT POSITIVE       MP041809
      IF (P.LE.0) RETURN                                                MP041811
C CHECK THAT ENOUGH SPACE IS AVAILABLE                                  MP041813
      CALL MPCHK (3, P+11)                                              MP041815
      I2 = 3*T + 12                                                     MP041817
C CONVERT X TO DECIMAL FORM                                             MP041819
      CALL MPOUT (X, R(I2), P, N)                                       MP041821
C AND WRITE ON UNIT LUN WITH FORMAT IFORM                               MP041823
      CALL MPIO (R(I2), P, LUN, IFORM, ERR)                             MP041825
      RETURN                                                            MP041827
      END                                                               MP041829

C $$                   ******  MPSIGA  ******                           MP048741
      FUNCTION MPSIGA (X)                                               MP048743
C RETURNS SIGN OF MP NUMBER X                                           MP048745
      INTEGER X(1)                                                      MP048747
      MPSIGA = X(1)                                                     MP048749
      RETURN                                                            MP048751
      END                                                               MP048753

C $$                   ******  MPSIGB  ******                           MP048761
      SUBROUTINE MPSIGB (I, X)                                          MP048763
C SETS SIGN OF MP NUMBER X TO I.                                        MP048765
C I SHOULD BE 0, +1 OR -1.                                              MP048767
C EXPONENT AND DIGITS OF X ARE UNCHANGED,                               MP048769
C BUT RESULT MUST BE A VALID MP NUMBER.                                 MP048771
      COMMON B, T, M, LUN, MXR, R                                       MP048773
```

```
      INTEGER B, T, R(1), X(3)                                          MP048775
      X(1) = I                                                          MP048777
C CHECK FOR VALID SIGN                                                  MP048779
      IF (IABS(I).LE.1) GO TO 20                                        MP048781
      WRITE (LUN, 10)                                                   MP048783
   10 FORMAT (39H *** INVALID SIGN IN CALL TO MPSIGB ***)               MP048785
      GO TO 40                                                          MP048787
C RETURN IF X ZERO                                                      MP048789
   20 IF (I.EQ.0) RETURN                                                MP048791
C CHECK FOR VALID EXPONENT AND LEADING DIGIT                            MP048793
      IF ((IABS(X(2)).LE.M).AND.(X(3).GT.0).AND.(X(3).LT.B)) RETURN     MP048795
      WRITE (LUN, 30)                                                   MP048797
   30 FORMAT (48H *** X NOT VALID MP NUMBER IN CALL TO MPSIGB ***)      MP048799
   40 CALL MPERR                                                        MP048801
      X(1) = 0                                                          MP048803
      RETURN                                                            MP048805
      END                                                               MP048807

C $$                   ******  MPUPK  ******                            MP052341
      SUBROUTINE MPUPK (SOURCE, DEST, LDEST, LFIELD)                    MP052343
C                                                                       MP052345
C               **************************                              MP052347
C               *** MACHINE DEPENDENT ***                               MP052349
C               **************************                              MP052351
C                                                                       MP052353
C MACHINE-DEPENDENT STATEMENTS ARE SURROUNDED BY C *** LINES            MP052355
C ***                                                                   MP052357
C THIS IS UNIVAC 1100, FORTRAN V VERSION.                               MP052359
C ***                                                                   MP052361
C THIS SUBROUTINE UNPACKS A PACKED HOLLERITH STRING (SOURCE)            MP052363
C PLACING ONE CHARACTER PER WORD IN THE ARRAY DEST (AS IF READ IN       MP052365
C A1 FORMAT). IT CONTINUES UNPACKING UNTIL IT FINDS A SENTINEL ($)      MP052367
C OR UNTIL IT FINDS A COMPILER GENERATED SENTINEL (IF SO                MP052369
C IMPLEMENTED) OR UNTIL IT HAS FILLED LDEST WORDS OF THE                MP052371
C ARRAY DEST.  THE LENGTH OF THE UNPACKED STRING IS RETURNED            MP052373
C IN LFIELD.   THUS 0 .LE. LFIELD .LE. LDEST.                           MP052375
      INTEGER SOURCE(1), DEST(1), BLANKS, TEMP                          MP052377
      DATA BLANKS /1H /, IST /1H$/                                      MP052379
C NK IS THE NUMBER OF CHARACTERS PER WORD                               MP052381
C AND ISTC IS THE COMPILER-GENERATED SENTINEL (IF ANY)                  MP052383
C ***                                                                   MP052385
      DATA NK /6/, ISTC /0/                                             MP052387
C ***                                                                   MP052389
      TEMP = BLANKS                                                     MP052391
      LD = LDEST                                                        MP052393
      LFIELD = 0                                                        MP052395
      IF (LD.LE.0) RETURN                                               MP052397
      DO 10 K = 1, LD                                                   MP052399
      I = LFIELD/NK + 1                                                 MP052401
C GET NEXT WORD (CONTAINING NK CHARACTERS) AND                          MP052403
C CHECK FOR COMPILER-GENERATED END-OF-STRING SENTINEL                   MP052405
      IF (SOURCE(I) .EQ. ISTC) RETURN                                   MP052407
C MOVE (MOD(LFIELD,NK)+1)-TH CHARACTER OF SOURCE(I) TO                  MP052409
C FIRST (I.E. LEFTMOST) CHARACTER POSITION OF TEMP                      MP052411
C ***                                                                   MP052413
      FLD (0, 6, TEMP) = FLD (6*MOD(LFIELD,6), 6, SOURCE(I))            MP052415
```

```
C ***                                                                MP052417
C CHECK FOR END-OF-STRING SENTINEL                                    MP052419
      IF (TEMP .EQ. IST)  RETURN                                      MP052421
      LFIELD = K                                                      MP052423
   10 DEST(K) = TEMP                                                  MP052425
      RETURN                                                          MP052427
      END                                                             MP052429

      SUBROUTINE MPUPK (SOURCE, DEST, LDEST, LFIELD)
C
C                ***************************
C                *** MACHINE DEPENDENT ***
C                ***************************
C
C MACHINE-DEPENDENT STATEMENTS ARE SURROUNDED BY C *** LINES
C ***
C THIS IS UNIVAC 1100, ASCII FORTRAN VERSION.
C ***
C THIS SUBROUTINE UNPACKS A PACKED HOLLERITH STRING (SOURCE)
C PLACING ONE CHARACTER PER WORD IN THE ARRAY DEST (AS IF READ IN
C A1 FORMAT). IT CONTINUES UNPACKING UNTIL IT FINDS A SENTINEL ($)
C OR UNTIL IT FINDS A COMPILER GENERATED SENTINEL (IF SO
C IMPLEMENTED) OR UNTIL IT HAS FILLED LDEST WORDS OF THE
C ARRAY DEST.  THE LENGTH OF THE UNPACKED STRING IS RETURNED
C IN LFIELD.   THUS 0 .LE. LFIELD .LE. LDEST.
      INTEGER SOURCE(1), DEST(1), BLANKS, TEMP
      DATA BLANKS /1H /, IST /1H$/
C NK IS THE NUMBER OF CHARACTERS PER WORD
C AND ISTC IS THE COMPILER-GENERATED SENTINEL (IF ANY)
C ***
      DATA NK /4/, ISTC /0/
C ***
      TEMP = BLANKS
      LD = LDEST
      LFIELD = 0
      IF (LD.LE.0) RETURN
      DO 10 K = 1, LD
      I = LFIELD/NK + 1
C GET NEXT WORD (CONTAINING NK CHARACTERS) AND
C CHECK FOR COMPILER-GENERATED END-OF-STRING SENTINEL
      IF (SOURCE(I) .EQ. ISTC) RETURN
C MOVE (MOD(LFIELD,NK)+1)-TH CHARACTER OF SOURCE(I) TO
C FIRST (I.E. LEFTMOST) CHARACTER POSITION OF TEMP
C ***
      BITS (TEMP, 1, 9) = BITS (SOURCE(I), 9*MOD(LFIELD,4)+1, 9)
C ***
C CHECK FOR END-OF-STRING SENTINEL
      IF (TEMP .EQ. IST)  RETURN
      LFIELD = K
   10 DEST(K) = TEMP
      RETURN
      END

C $$                  ******  MPUPK  ******
      SUBROUTINE MPUPK (SOURCE, DEST, LDEST, LFIELD)                  MP052341
C                                                                     MP052343
                                                                      MP052345
```

```
C             ***************************                             MP052347
C             *** MACHINE DEPENDENT ***                               MP052349
C             ***************************                             MP052351
C                                                                     MP052353
C MACHINE-DEPENDENT STATEMENTS ARE SURROUNDED BY C *** LINES           MP052355
C ***                                                                 MP052357
C THIS IS IBM 360 FORTRAN G OR H VERSION
C ***                                                                 MP052361
C THIS SUBROUTINE UNPACKS A PACKED HOLLERITH STRING (SOURCE)          MP052363
C PLACING ONE CHARACTER PER WORD IN THE ARRAY DEST (AS IF READ IN     MP052365
C A1 FORMAT). IT CONTINUES UNPACKING UNTIL IT FINDS A SENTINEL ($)    MP052367
C OR UNTIL IT FINDS A COMPILER GENERATED SENTINEL (IF SO              MP052369
C IMPLEMENTED) OR UNTIL IT HAS FILLED LDEST WORDS OF THE              MP052371
C ARRAY DEST.  THE LENGTH OF THE UNPACKED STRING IS RETURNED          MP052373
C IN LFIELD.   THUS 0 .LE. LFIELD .LE. LDEST.                         MP052375
      INTEGER DEST(1), BLANKS, TEMP
C ***
      LOGICAL*1 SOURCE(1), TC(4)
      EQUIVALENCE (TC, TEMP)
C ***
      DATA BLANKS /1H /, IST /1H$/                                    MP052379
C NK IS THE NUMBER OF CHARACTERS PER WORD                             MP052381
C AND ISTC IS THE COMPILER-GENERATED SENTINEL (IF ANY)                MP052383
C ***                                                                 MP052385
      DATA NK /4/, ISTC /0/
C ***                                                                 MP052389
      TEMP = BLANKS                                                   MP052391
      LD = LDEST                                                      MP052393
      LFIELD = 0                                                      MP052395
      IF (LD.LE.0) RETURN                                             MP052397
      DO 10 K = 1, LD                                                 MP052399
C ***                                                                 MP052413
      TC(1) = SOURCE (K)
C ***                                                                 MP052417
C CHECK FOR END-OF-STRING SENTINEL                                    MP052419
      IF (TEMP .EQ. IST)  RETURN                                      MP052421
      LFIELD = K                                                      MP052423
   10 DEST(K) = TEMP                                                  MP052425
      RETURN                                                          MP052427
      END                                                             MP052429
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

We describe the procedure required to interface the FORTRAN multiple precision package of Richard P. Brent (as described in ACM Transactions on Mathematical Software, March, 1978) with the AUGMENT precompiler for FORTRAN. We also indicate the method of using the multiple precision arithmetic package in conjunction with AUGMENT.